

## MINISYMPOSIUM

## A MULTISCALE APPROACH TO 'GROWTH AND FORM'

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Despite the passing of a 100 years since the publication of Darcy Thompson's seminal treatise 'On Growth and Form' the majority of phenomena that shape organisms and their environment remain at best only partially understood. This is in part due to the fact that the processes involved occur over a multitude of scales from phenomena occurring within a single cell to tissue level or even population level effects. Mathematics in the form of modelling, analysis and simulation plays a central role in describing and understanding such multiscale phenomena.

In this minisymposia we focus on the derivation, analysis and simulation of PDE models that couple phenomena occurring over multiple scales to explain biological processes. The biological problems range from invasion in ecology to tissue growth. The mathematics involved ranges from the derivation of rational models using theoretical mechanics to homogenisation and free boundary problems and advanced numerical techniques for the simulation of multiscale PDEs on complex deforming geometries. The overall goal is to bring together experts on the derivation, analysis and simulation of multiscale PDE models in biology to report on recent developments and to illustrate the applications of mathematical techniques to better understand biological problems as well as to discuss future directions.

*Minisymposium: A Multiscale approach to 'Growth and Form'*

# MULTIPHASE APPROACHES TO TISSUE GROWTH

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*Keywords:* Tissue growth, Multiphase models.

A hierarchy of PDE models for tissue growth and mechanics will be described and some of their properties outlined.

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## EVOLUTION BY COMPETITION-DIFFUSION IN HETEROGENEOUS ENVIRONMENTS

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Joint work with Chandrasekhar Venkataraman (St Andrews, UK)

*Keywords:* Free boundary problems, Strong competition, Heterogeneous diffusion.

In most of the models in theoretical ecology one assumes that competing individuals experience the same environments. A more realistic scenario would be to study such models in heterogeneous environments. In this talk, I will be attempting to give an overview of a recent work which provides a starting point towards understanding the role played by heterogeneous motility on invasion behaviour in mathematical models for competition between motile species. The results that I present are a joint work with C. Venkataraman some of which can be found in [1].

We have studied the effect of rapidly oscillating periodic motilities in a competition-diffusion system for the dynamics of two interacting species while performing simultaneous homogenization and strong competition limits. The limit problem is shown to be a free boundary problem of Stefan type with effective coefficients. We have also performed some numerical simulations in one and two spatial dimensions that suggest that oscillations in the motilities are detrimental to invasion behaviour of a species.

In this talk, I will also be commenting on scenarios where the interspecific competition rates and the oscillation frequencies in the motilities are of different orders. Mathematically, this corresponds to performing asymptotic analysis while two parameters vanish independently. It is well-known that, in contrast with the periodic case, the presence of lower order terms in parabolic problems with random coefficients might drastically change the effective behaviour of solutions. This talk will also touch upon some results when the motility coefficients are periodic in the spatial variables and random in time.

*Minisymposium: A Multiscale approach to 'Growth and Form'*

## HOW TO SHAPE AN ORGAN? - COMPUTATIONAL MODELS OF ORGANOGENESIS

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*Keywords:* Organogenesis, Pattern formation, Lewis' law.

Animals develop from a single cell. While much is known about the regulatory programs that control development, it is still an open question how size and shape are determined in a growing animal. In my talk, I will present a novel mechanism for size control and growth termination. During growth, patterns emerge that define different parts of the developing animal. I will discuss how scaled patterning can be achieved on growing domains with a classical threshold-based French Flag read-out mechanism. The branched trees of lungs, kidneys and many glands provide a fascinating example of complex shape formation. I will show how the very different branching patterns in lungs and kidneys can robustly emerge from the same regulatory mechanism - implemented by very different protein families in the different organs. Finally, I will discuss the role of mechanics in epithelial organisation and the origin of Lewis' law.

*Minisymposium: A Multiscale approach to 'Growth and Form'*

## CELL-BASED AND CONTINUUM SCALE MODELS FOR THE CONTRACTION OF BURNS

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*Keywords:* Burn contraction, Cell migration.

Serious burn injuries are often characterised by the occurrence of contractures and the development of hypertrophic scar tissue, in which the first-mentioned complication may cause loss of mobility to the patient. In this talk, we will review cell-scale models used on the microscopic scale of the skin, as well as fully continuum-scale models, which are used to simulate larger domains of skin tissue. The models take into account fibroblasts that produce collagen, as well as myofibroblasts which exert large inward forces to the environment. These inward forces cause the contraction of the wound. Whereas the cell-scale models are based on stochastic processes for the appearance of cells, differentiation, death and proliferation and even migration of the cells, the continuum-scale models possess a deterministic nature. The uncertainty in the input data as a result of patient-specific values requires the use of (stochastic) parameter variation, which will also enable the prediction of the likelihood that a severe contraction takes place.

## References

- [1] H. Hutridurga and C. Venkataraman. (2017). *Heterogeneity and strong competition in ecology*, ArXiv preprint. arXiv:1710.06116